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TOPOLOGY OPTIMIZATION IN FRACTURE MECAHNICS USING THE LEVEL-SET METHOD

Jeet DESAI[1,3]

Grégoire ALLAIRE[2], François JOUVE[3]

[1] Institut de Recherche Technologique SystemX [2] Centre de Mathématiques Appliquées, École Polytechnique [3] Laboratoire J.L. Lions, Université de Paris

Abstract: **We are interested in the topology optimization in fracture mechanics. We consider a linear elastic structure subjected to fracture, modelled by a damage model. The rigidity of the structure is maximized under the constraint of volume.**

1 GAP

- Theory of shape optimization developed for partial differential equations, in particular, linear elasticity
- Fracture mechanics being governed by inequation remains a subject of research

2 FRACTURE MODEL

By using the damage model [1]

- Parameter $\alpha : \Omega \mapsto [0,1]$
- Hooke's Tensor: $\mathbb{C}(\alpha) = (1-\alpha)^2\mathbb{C}$
- Minimization of the mechanical energy

$$\mathcal{W}_l(\varepsilon(\mathbf{u}), \alpha, \nabla\alpha) = \frac{1}{2}\mathbb{C}(\alpha)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) + w(\alpha) + \frac{1}{2}w_1 l^2 \nabla\alpha \cdot \nabla\alpha$$

(where l is the characteristic length) under the constraint $\dot{\alpha} > 0$.

3 PROPOSITION

- We penalize the region: $\dot{\alpha} < 0$, $\alpha < 0$ with $0 < \epsilon \ll 1$
- $Z = H_0^1(\Omega)^d \times L^2(\Omega)$, $d = 2,3$
- We consider a time interval $[0,T]$ and find $(\mathbf{u}, \alpha) \in Z$ such that $\forall(\boldsymbol{\varphi}, \psi) \in Z$

$$\int_\Omega \mathbb{C}(\alpha)\varepsilon(\mathbf{u}) : \varepsilon(\boldsymbol{\varphi})\, dx = \int_{\Gamma_N} \mathbf{g} \cdot \boldsymbol{\varphi}\, ds \qquad (1)$$

where $\mathbf{g}$ is the applied force and

$$\int_\Omega \frac{\partial \mathcal{W}_l(\mathbf{u}, \alpha)}{\partial \alpha}\psi\, dx + \frac{1}{\epsilon}\Big(\int_\Omega \max(\alpha - 1, 0)\psi - \max(-\dot{\alpha}, 0)\psi\Big) dx = 0 \quad (2)$$

4 OPTIMIZATION OBJECTIVE

We maximize the elastic energy integrated in time

$$J(\mathbf{u}, \alpha, \Omega) = -\int_0^T \int_\Omega \mathbb{C}(\alpha)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) dx\, dt$$

under the volume constraint $\int_\Omega dx \leq V_t$ with applied displacement.

5 SHAPE DESCRIPTION

- By a level-set function $\phi : \mathbb{R}^d \to \mathbb{R}$: [2]

$$\begin{cases} \phi(x) < 0 \text{ if } x \in \Omega, \\ \phi(x) = 0 \text{ if } x \in \partial\Omega, \\ \phi(x) > 0 \text{ if } x \in \overline{\Omega}^c. \end{cases}$$

- By remeshing the domain described implicitly by level-set [3].

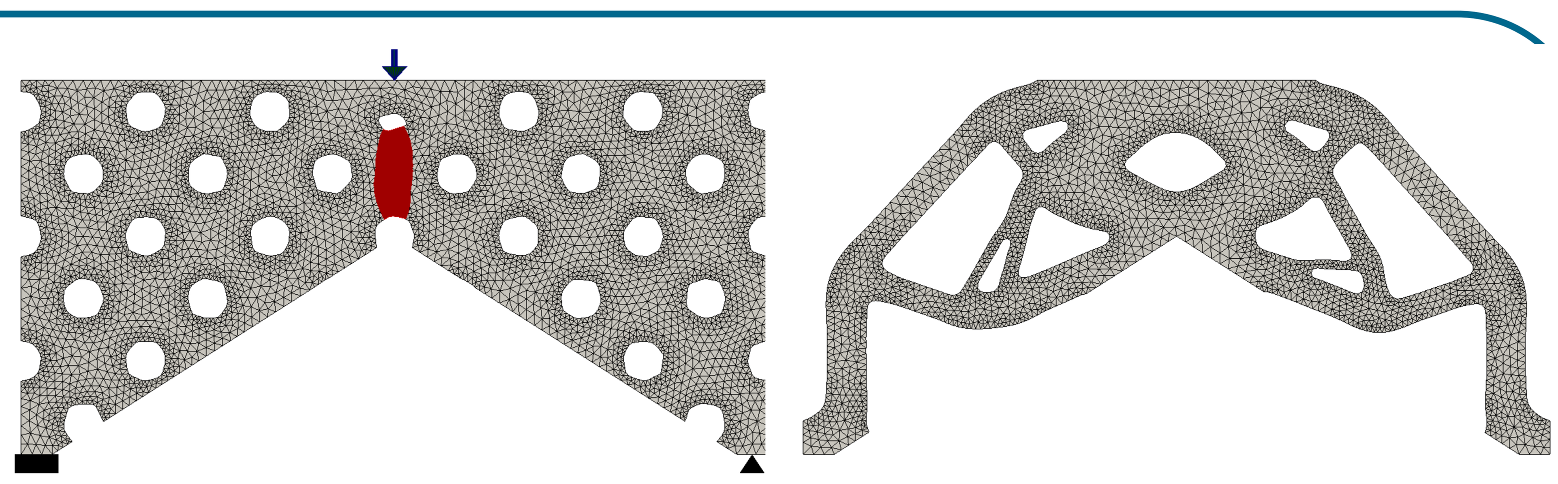
6 SHAPE DERIVATIVE

We deform the domain Ω along the direction of the shape derivative $\nabla J(\mathbf{u}, \alpha, \Omega)$ given by

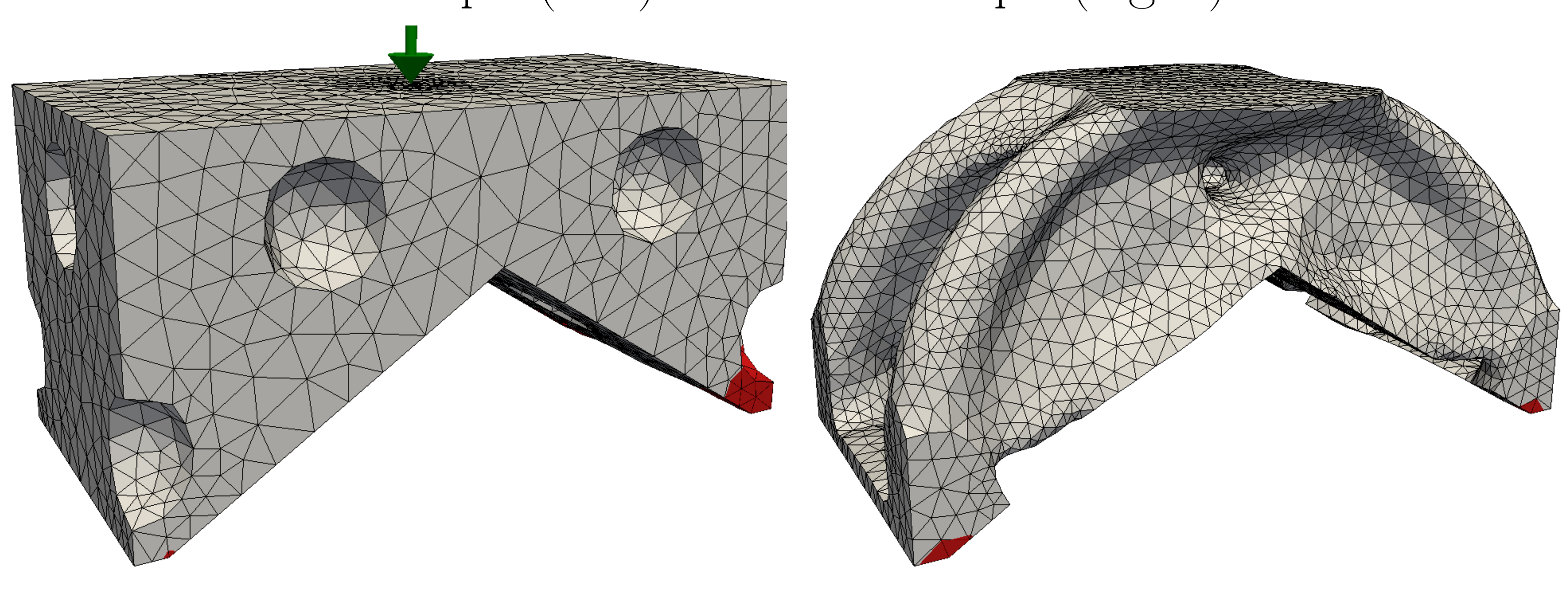
$$\nabla J(\mathbf{u}, \alpha, \Omega) = \Big(-\mathbb{C}(\alpha)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) + \frac{1}{2}\mathbb{C}(\alpha)'\beta\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) + \mathbb{C}(\alpha)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{v}) + w'(\alpha)\beta + \frac{1}{2}w_1 l^2 \nabla\alpha \cdot \nabla\beta\Big)$$

where $(\mathbf{u}, \alpha)$ is the solution of (1)-(2) and $(\mathbf{v}, \beta)$ is the adjoint solution.

7 RESULTS



2D example: Mesh and the damaged region (in red); Initial shape (left) and final shape (right)



3D example: Mesh and the damaged region (in red); Initial shape (left) and final shape (right)

Références

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Université de Paris
Jeet Desai- jeet.Desai@irt-systemx.fr