

# Advances in Learning with Bayesian Networks

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DUKe (Data User Knowledge) research group, LS2N UMR 6004, Nantes, France

Séminaire IRT SystemX, 19 jan. 2017



# LS2N - UMR CNRS 6004

1er janvier 2017, fusion de 2 UMR

- IRCCyN (Institut de Recherche en Communications et Cybernétique de Nantes)
- LINA (Laboratoire d'Informatique de Nantes Atlantique)
- 450 personnes ( $\approx$  215 permanents)
- 5 axes thématiques
  - Conception et Conduite de Systèmes
  - Robotique, Procédés, Calcul
  - Signaux, Images, Ergonomie et Langues
  - Science du Logiciel et des Systèmes Distribués
  - Science des Données et de la Décision
- 5 établissements support
  - Ecole Centrale de Nantes
  - Université de Nantes
  - Institut Mines Telecom Atlantique
  - CNRS, INRIA



## DUKe research group

### Research group in Data Sciences

- 16 permanents
- 3 associates
- 12 PhD students
- 2 postdoc/engineers

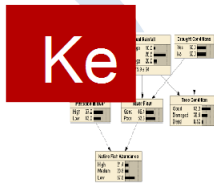
# DUKe scientific approach

Proposing « agile », « user in the loop »  
 Data-mining / Machine Learning algorithms

User



Data



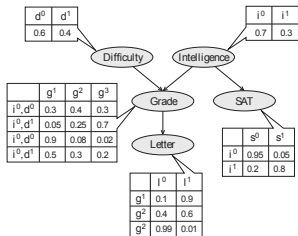
Knowledge



Applications and valorisation  
 Digital Humanity, BioInformatics,  
 Business Intelligence

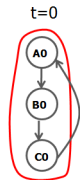
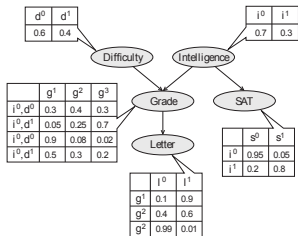
# Motivations

- Bayesian networks (BNs) are a powerful tool for graphical representation of the underlying knowledge in the data and reasoning with incomplete or imprecise observations.
- BNs have been extended (or generalized) in several ways, as for instance, causal BNs, dynamic BNs, relational BNs, ...

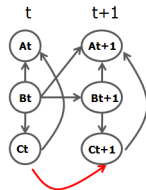


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Initial Model



Transition Model

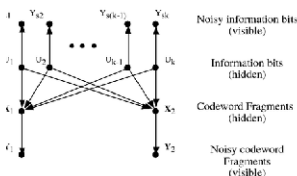
# Motivations

## Victim identification system



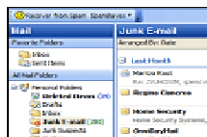
G\_fa: Genotype father  
 G\_mo: Genotype mother  
 G\_chi: Genotype child  
 A\_fa: Paternal Allele  
 A\_mo: Maternal Allele

## Turbo-codes (GSM, ...)



Noisy information bits (visible)  
 Information bits (hidden)  
 Codeword Fragments (hidden)  
 Noisy codeword Fragments (visible)

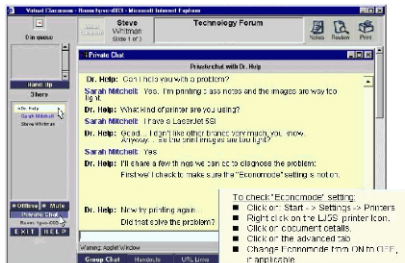
## Anti Spam



## Assistant iPhone SIRI



## After-sale services



## MS Office assistant

It looks like you're writing a letter.

Would you like help?

- Get help with writing the letter
- Just type the letter without help

Don't show me this tip again







# Motivations

We would like to learn a BN from data... but which kind of data ?

- complete /incomplete [François 06]



A	B	C	D
0	1	2	3
4	?	1	0
2	3	5	?
...			
...			
...			
1	3	?	6
3	8	9	0
1	2	4	?
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- high  $n$ ,

- 
- 
- 
- 

A	B	C	D	...	...	...	X <sub>100000</sub>
0	1	2	3	...	...	...	7
4	6	1	0	...	...	...	5
2	3	5	6	...	...	...	4
			...				...
			...				...
			...				...
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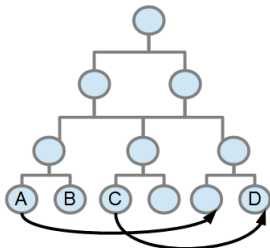
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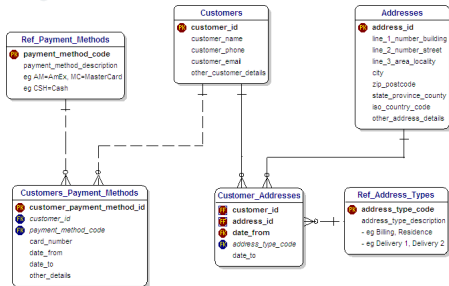
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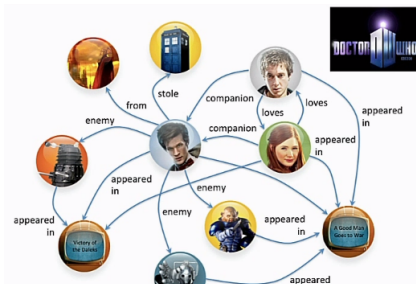
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- structured data [Ben Ishak 15, Coutant 15, Chulyadyo 16]



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- structured data [Ben Ishak 15, Coutant 15, Chulyadyo 16]
- not so structured data [Elabri]



# Motivations

Even the learning task can differ : generative

- modeling  $P(X, Y)$
- no target variable
- more general model
- better behavior with incomplete data

## Objectives of this talk

- how to learn BNs in such various contexts ?
- state of the art : founding algorithms and recent ones
- pointing out **our contributions** in this field



# Motivations

Even the learning task can differ : generative vs. discriminative

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- modeling  $P(Y|X)$
- one target variable  $Y$
- dedicated model

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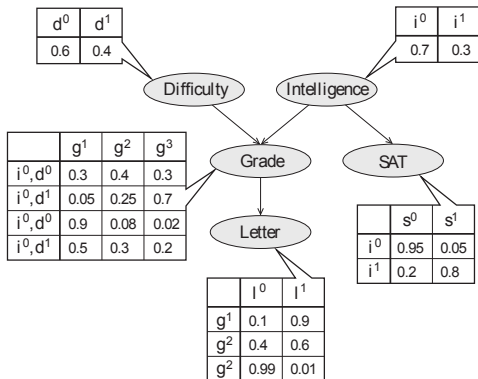
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# Bayesian network

[Pearl, 1985]

## Definition

- G qualitative description of conditional dependences / independences between variables  
**directed acyclic graph (DAG)**
- ⊖ quantitative description of these dependences  
**conditional probability distributions (CPDs)**



## Main property

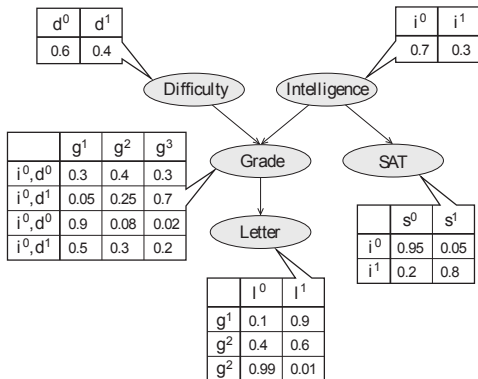
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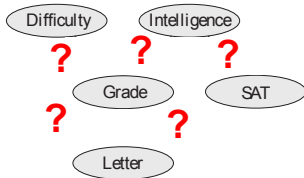




# One model... but two learning tasks

**BN = graph  $G$  and set of CPDs  $\Theta$**

- parameter learning /  $G$  given
- structure learning





# Parameter learning (generative)

## Complete data $\mathcal{D}$

- max. of likelihood (ML) :  $\hat{\theta}^{MV} = \operatorname{argmax} P(\mathcal{D}|\theta)$
- closed-form solution :

$$\hat{P}(X_i = x_k | Pa(X_i) = x_j) = \hat{\theta}_{i,j,k}^{MV} = \frac{N_{i,j,k}}{\sum_k N_{i,j,k}}$$

$N_{i,j,k}$  = nb of occurrences of  $\{X_i = x_k \text{ and } Pa(X_i) = x_j\}$

## Other approaches

$P(\theta) \sim \text{Dirichlet}(\alpha)$

- max. a posteriori (MAP) :  $\hat{\theta}^{MAP} = \operatorname{argmax} P(\theta|\mathcal{D})$
- expectation a posteriori (EAP) :  $\hat{\theta}^{EAP} = E(P(\theta|\mathcal{D}))$

$$\hat{\theta}_{i,j,k}^{MAP} = \frac{N_{i,j,k} + \alpha_{i,j,k} - 1}{\sum_k (N_{i,j,k} + \alpha_{i,j,k} - 1)}$$

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# Parameter learning (generative)

## Incomplete data

- no closed-form solution
- EM (iterative) algorithm [Dempster, 77], convergence to a local optimum

## Incremental data

- advantages of sufficient statistics

$$\theta_{i,j,k} = \frac{N^{old} \theta_{i,j,k}^{old} + N_{i,j,k}}{N^{old} + N}$$

- this Bayesian updating can include a forgetting factor

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# BN structure learning is a complex task

## Size of the "solution" space

- the number of possible DAGs with  $n$  variables is super-exponential w.r.t  $n$  [Robinson, 77]

$$NS(5) = 29281 \quad NS(10) = 4.2 \times 10^{18}$$

- an exhaustive search is impossible for realistic  $n$  !

One thousand millenniums =  $3.2 \times 10^{13}$  seconds

## Identifiability

- data can only help finding (conditional) dependences / independences
- Markov Equivalence : several graphs describe the same dependence statements
- causal Sufficiency : do we know all the explaining variables ?

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# Structure learning (generative / complete)

## Constraint-based methods

- BN = independence model  
⇒ find CI in data in order to build the DAG  
ex : IC [Pearl & Verma, 91], PC [Spirtes et al., 93]
- problem : reliability of CI statistical tests (ok for  $n < 100$ )

## Score-based methods

- BN = probabilistic model that must fit data as well as possible
- problem : size of search space (ok for  $n < 1000$ )

## Hybrid/ local search methods

- local search / neighbor identification (statistical tests)
- global (score) optimization
- usually for scalability reasons (ok for high  $n$ )

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## Score-based methods

- BN = probabilistic model that must fit data as well as possible  
⇒ search the DAG space in order to maximize a scoring function  
ex : Maximum Weighted Spanning Tree [Chow & Liu, 68], Greedy Search [Chickering, 95], evolutionary approaches [Larranaga et al., 96] [Wang & Yang, 10]
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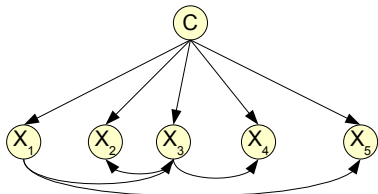
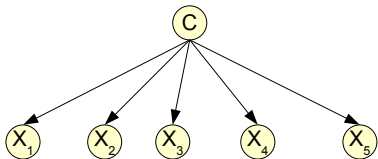
## Hybrid/ local search methods

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- global (score) optimization
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- ex : MMHC algorithm [Tsamardinos et al., 06]

# Structure learning (discriminative)

## Specific structures

- naive Bayes, augmented naive Bayes
- multi-nets
- ...



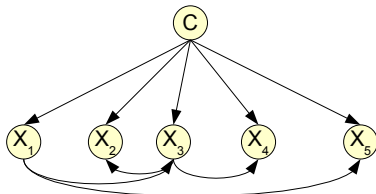
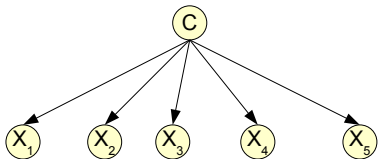
## Structure learning

- usually, the structure is learned in a generative way
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## Structure learning

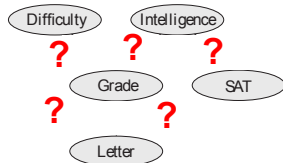
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# Structure learning

## Incomplete data

- hybridization of previous structure learning methods and EM
- ex : Structural EM [Friedman, 97]  $\simeq$  Greedy Search + EM
- problem : convergence

A	B	C	D
0	1	2	3
4	?	1	0
2	3	5	?
...			
...			
...			
1	3	?	6
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# Structure learning

$n \gg p$

- robustness and complexity issues
- application of Perturb & Combine principle
- ex : mixture of randomly perturbed trees  
[Ammar & Leray, 11]

A	B	C	D	...	...	...	$X_{100000}$
0	1	2	3	...	...	...	7
4	6	1	0	...	...	...	5
2	3	5	6	...	...	...	4

# Structure learning

## Incremental learning and data streams

- Bayesian updating is easy for parameters
- Bayesian updating is complex for structure learning
- and other constraints related to data streams (limited storage, ...)
- ex : incremental MMHC  
[Yasin and Leray, 13]

A	B	C	D	...	...	...	$X_{100000}$
0	1	2	3	...	...	...	7
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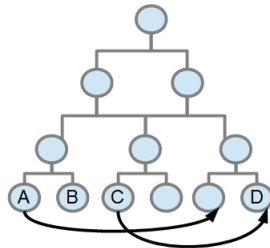


# Structure learning

## Integration of prior knowledge

- in order to reduce search space : white list, black list, node ordering [Campos & Castellano, 07]
- interaction with ontologies [Ben Messaoud et al., 13]

A	B	C	D
0	1	2	3
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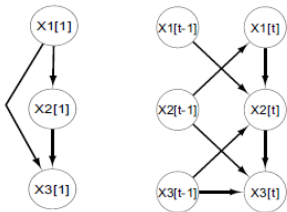
# Dynamic Bayesian networks (DBNs)

## **k slices temporal BN (k-TBN)** [Murphy, 02]

- $k - 1$  Markov order
- prior graph  $G_0$  + transition graph  $G_{\rightarrow}$
- for example : 2-TBNs model [Dean & Kanazawa, 89]

## Simplified k-TBN

- k-TBN with only temporal edges [Dojer, 06][Vinh et al, 12]



(a) Prior network (b) Transition network

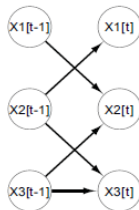
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(c) Transition network with only inter time-slice arcs

# DBN structure learning (generative)

## Score-based methods

- dynamic Greedy Search [Friedman et al., 98], genetic algorithm [Gao et al., 07], dynamic Simulated Annealing [Hartemink, 05], ...
- for k-TBN ( $G_0$  and  $G_{\rightarrow}$  learning)
- but not scalable (high  $n$ )

## Hybrid methods

- [Dojer, 06] [Vinh et al., 12] for simplified k-TBN, but often limited to  $k = 2$  for scalability
- dynamic MMHC for "unsimplified" 2-TBNs with high  $n$  [Trabelsi et al., 13]

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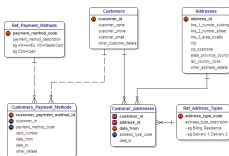


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## Relational DB

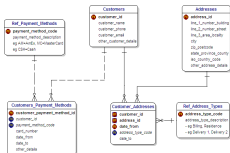
- Relational schema is given
- Learning prob. dep. between variables, but more complex !

# Motivations



## Flat data

- No relational model
- Learning probabilistic dependencies between variables



## Relational DB

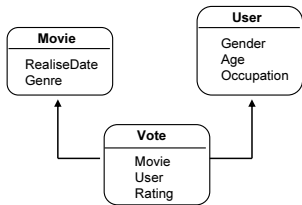
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## Graph DB

- Relational schema ?
- Learning prob. dep. between variables ?

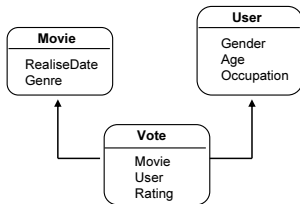
# Relational schema



## relational schema $\mathcal{R}$

- classes + attributes
- reference slots (e.g.  $Vote.Movie$ ,  $Vote.User$ )
- inverse reference slots (e.g.  $User.User^{-1}$ )
- slot chain = a sequence of (inverse) reference slots

# Relational schema

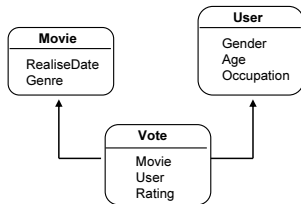


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• ex.  $Vote.User.User^{-1}.Movie$ : all the movies voted by a particular user

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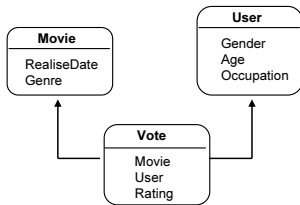


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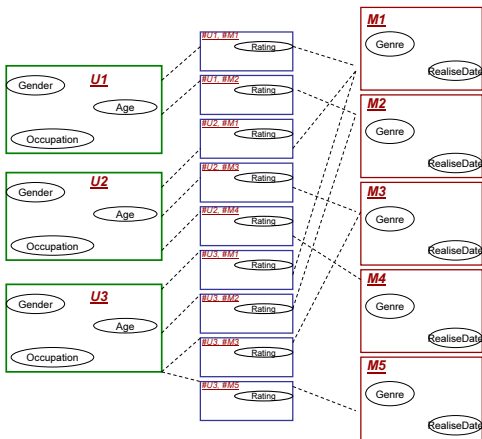
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# Relational skeleton



## Instance $\mathcal{I}$

- set of objects for each class
- with a value for each reference slot and each attribute
- == a "populated" database

## Relational skeleton $\sigma_{\mathcal{R}}$

- Instance without attribute values

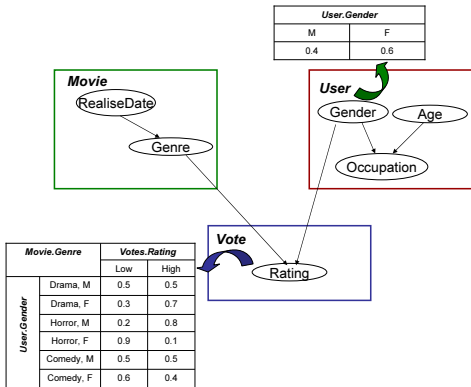
# Probabilistic Relational Models

[Koller & Pfeffer, 98]

## Definition

A PRM  $\Pi$  associated to  $\mathcal{R}$ :

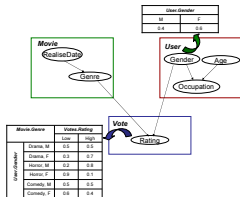
- a qualitative dependency structure  $\mathcal{S}$  (with possible long **slot chains** and **aggregation functions**)
- a set of parameters  $\theta_{\mathcal{S}}$





# Probabilistic Relational Models

## Definition



## Aggregators

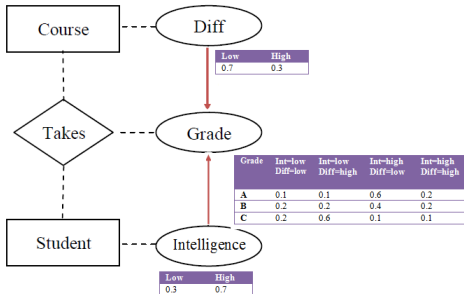
- $Vote.User.User^{-1}.Movie.genre \rightarrow Vote.rating$
- movie rating from one user can be dependent with the genre of all the movies voted by this user
  - how to describe the dependency with an unknown number of parents ?
  - solution : using an aggregated value, e.g.  $\gamma = MODE$

# DAPER

Another probabilistic relational model [Heckerman & Meek, 04]

## Definition

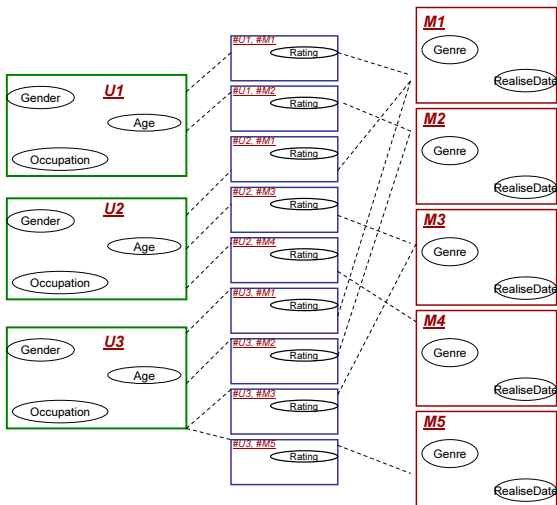
- Probabilistic model associated to an Entity-Relationship model
- Classes = { Entity classes + Relationship classes }



# Learning from a relational dataset

## PRM/DAPER learning

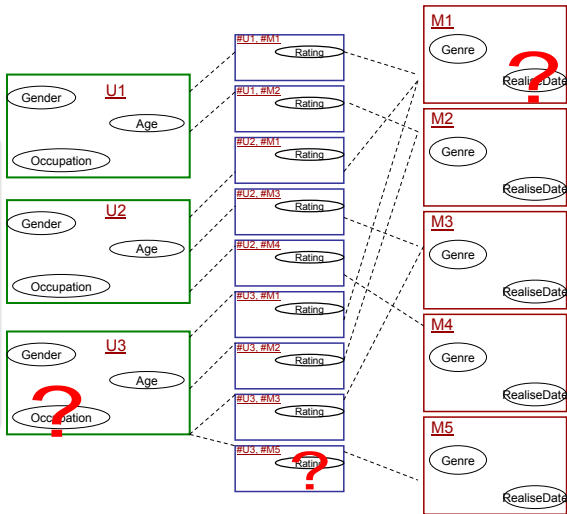
- finding the probabilistic dependencies and the probability tables from an instantiated database
- relational schema is known, but ...
- several situations / PRM extensions



# Learning from a relational dataset

## Attribute uncertainty

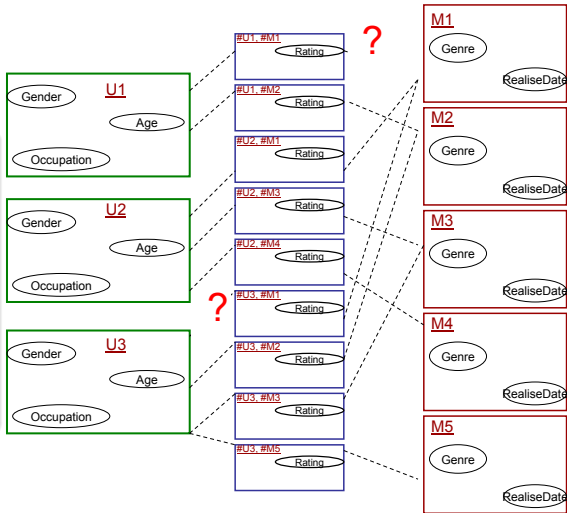
- Input : relational skeleton (all the objects and relations), some attributes
- Objective : predict only missing attributes



# Learning from a relational dataset

## Reference uncertainty

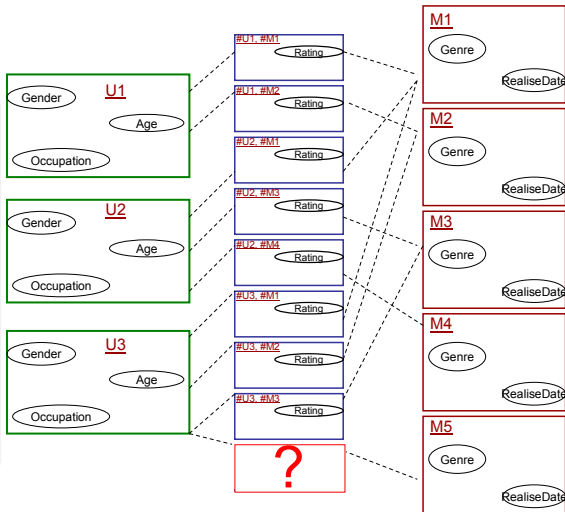
- Input : partial relational skeleton (all the objects, but some relations are missing)
- Objective : predict missing attributes and "foreign keys"



# Learning from a relational dataset

## Existence uncertainty

- Input : partial relational skeleton (all the entity objects, but some relationship objects are missing)
- Objective : predict existence of relationships between entity objects



# PRM/DAPER learning with AU

## Relational variables

- finding new variables by exploring the relational schema
  - ex: student.reg.grade, registration.course.reg.grade, registration.student reg.course.reg.grade, ...
- ⇒ adding another dimension in the search space
- ⇒ limitation to a given maximal slot chain length

## Constraint-based methods

- relational PC [Maier et al., 10] relational CD [Maier et al., 13]
- don't deal with aggregation functions

## Score-based methods

- Greedy search [Getoor et al., 07]

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# PRM/DAPER learning with RU

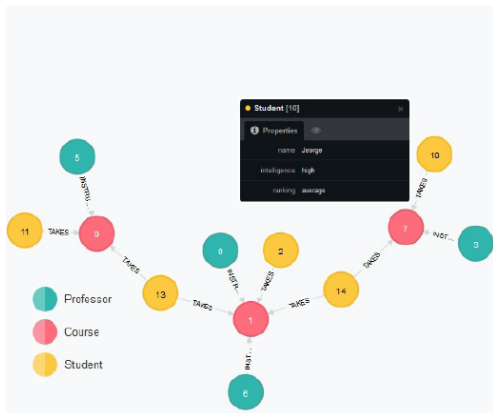
## Need for partitionning

- The missing foreign key is considered as a random variable
- We need to partition the similar "target" objects in order to obtain a generic model

## How to partition

- With object attributes [Getoor et al.] = clustering
- With relational information = graph partitionning
- With both : [Coutant et al., 15]

# Graph database

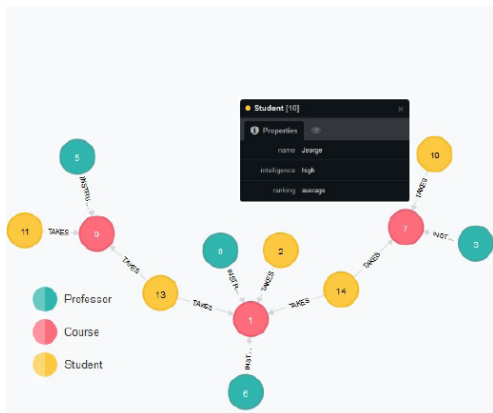


## Definition

- Data is described in a graph, with nodes and relationships
- Attributes can be associated to both.

## Properties

# Graph database

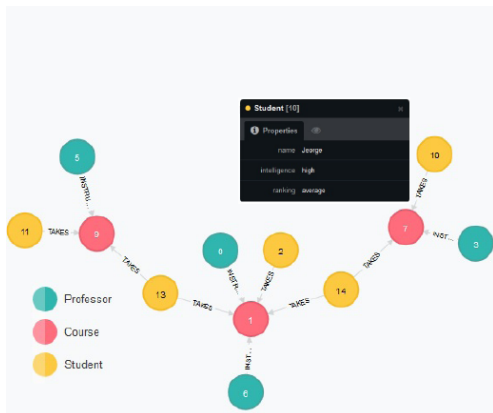


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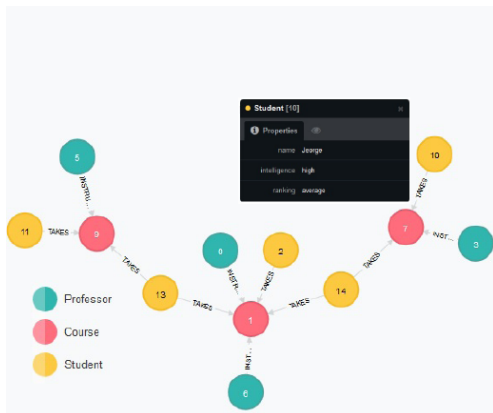


## Definition

## Properties

- Scalability / large data (no join operation, only graph traversal)
- Schema-free, no relational schema

# Graph database



## Definition

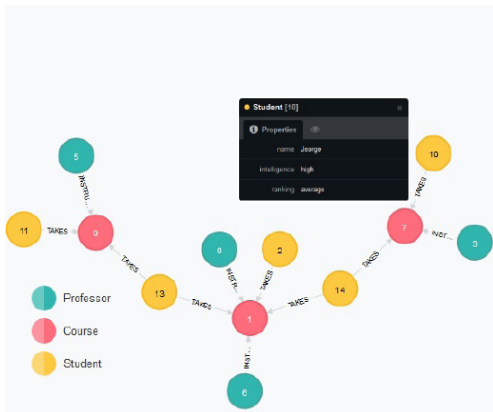
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# Learning from a Graph database

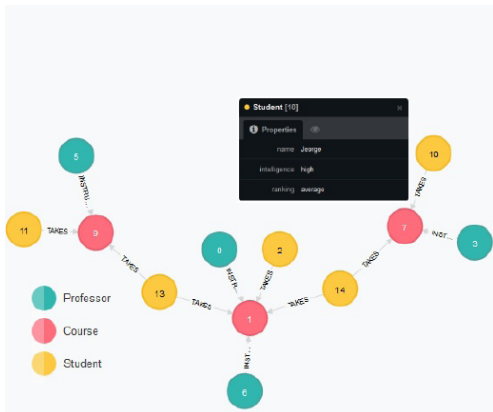


## Our assumptions

- Data is "organized" / stored by approx. following some meta/ER model.
- Use of labels in order to "type" nodes and relationships
- Otherwise, we can't do anything !

[Elabri, in progress]

# Learning from a Graph database



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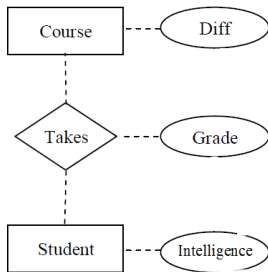
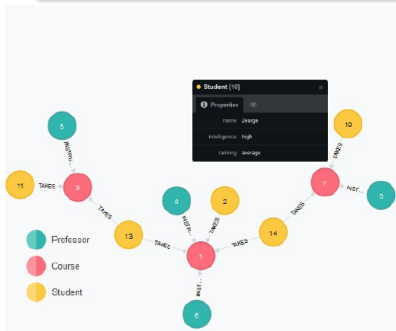
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# DAPER learning

## ER identification from data

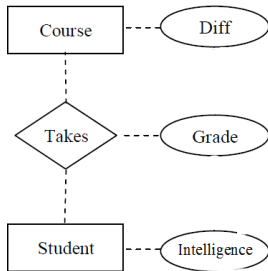
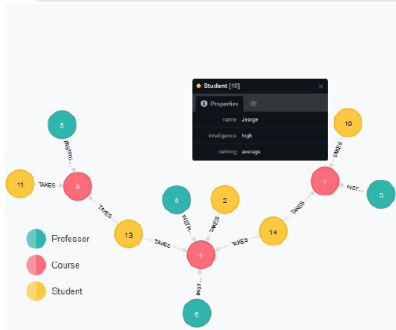
- E=node labels, R=relationship labels
- choosing only the most frequent signature  $(E_i \times E_j)$  for each R



# DAPER learning

## ER identification from data

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# DAPER learning

## DAPER structure learning

Once ER model is identified, we can learn the probabilistic dependencies :

- Attribute uncertainty : predicting attribute value only
- Reference uncertainty : predicting the target node for an existing relation ?
- Existence uncertainty : predicting a relationship between two existing nodes ?

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# Outline ...



- 1 **BN learning**
  - Definition
  - Parameter learning
  - Structure learning
- 2 **Dynamic BN learning**
  - Definition
  - Learning
- 3 **Relational BN learning**
  - Definitions
  - Learning with a relational DB
  - Learning with a Graph DB
- 4 **The end**



## Conclusion

### Visible face of this talk

- Bayesian networks = powerful tool for knowledge representation and reasoning with data
- Our contributions about BN learning in several contexts

Todo list, in progress

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- Interacting with some probabilistic & logic frameworks
- Implementation in our software platform **PILGRIM**

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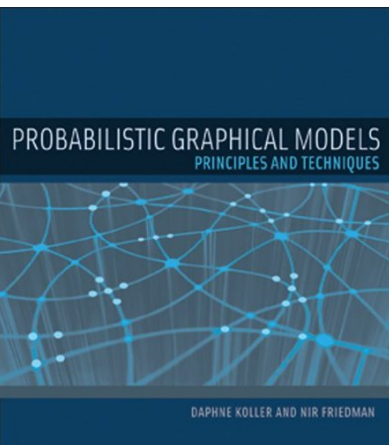
# One buzzword for the road

## BN Learning wrt. 7 Vs of Big Data

- **Volume** : scalable algorithms and map-reduce implementations
- **Variety** : flat data, SQL, graph databases, ...
- **Velocity/Variability** : incremental anytime learning, non stationary data
- **Visualization** : for user interaction
- **Veracity** : does the user give accurate data ?
- **Value** : of data ...



# References



## One starting point

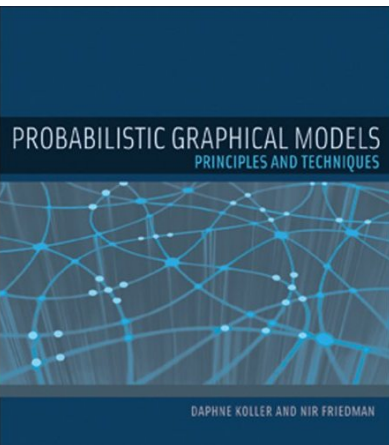
- [Koller & Friedman, 09]  
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Thank you for your  
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